Factors and Divisibility (Part 2—Required for Honors)

Section 1: Factor quadratics by grouping (KA link)

1. Factor $3x^2 - 2x - 1$ completely.

$3x^2 - 2x - 1$
$3x^2 - 3x + x - 1$
$3x(x - 1) + 1(x - 1)$
$(3x + 1)(x - 1)$

2. Factor $-3x^2 + 19x - 28$ completely.

$-3x^2 + 19x - 28$
$-3x^2 + 12x + 7x - 28$
$-3x(x - 4) + 7(x - 4)$
$(-3x + 7)(x - 4)$
$-(3x - 7)(x - 4)$

Section 2: Factor polynomials: quadratic methods (KA link)

1. Factor $p^2 + 5pq - 14q^2$ completely.

$-2 \cdot 7 = -14$ and $-2 + 7 = 5$

$p^2 + 5pq - 14q^2$
$p^2 + 7pq - 2pq - 14q^2$
$p(p + 7q) - 2q(p + 7q)$
$(p + 7q)(p - 2q)$

2. Factor $-5r^4 - 5r^3 + 60r^2$ completely.

$-5r^4 - 5r^3 + 60r^2 = -5r^2\left(r^2 + r - 12\right) = -5r^2(r + 4)(r - 3)$

3. Factor $q^{16} - 8q^8 + 15$ completely.

$q^{16} - 8q^8 + 15 = \left(q^8\right)^2 - 8\left(q^8\right) + 15$ Let $x = q^8$

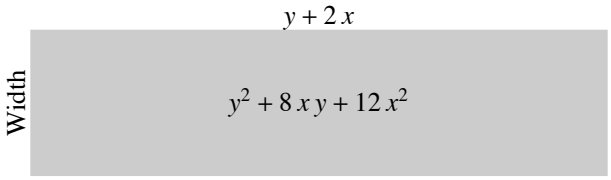
$x^2 - 8x + 15 = (x - 5)(x - 3) = \left(q^8 - 5\right)\left(q^8 - 3\right)$

Section 3: Factor polynomials: quadratic methods (challenge) (KA link)

1. Factor $a^2 + 3\,a - 2\,a\,b - 6\,b$ as the product of two binomials.

$a^2 + 3\,a - 2\,a\,b - 6\,b$
$a(a + 3) - 2\,b(a + 3)$
$(a + 3)\,(a - 2\,b)$

2. The rectangle below has an area of $y^2 + 8\,x\,y + 12\,x^2$ square meters and a length of $y + 2\,x$ meters. What expression represents the width of the rectangle?



$y^2 + 8\,x\,y + 12\,x^2$
$y^2 + 6\,x\,y + 2\,x\,y + 12\,x^2$
$y(y + 6\,x) + 2\,x(y + 6\,x)$
$(y + 6\,x)\,(y + 2\,x)$

$$A = l \cdot w \implies w = \frac{A}{l} = \frac{(y + 6\,x)\,(y + 2\,x)}{y + 2\,x} = y + 6\,x$$

Section 4: Difference of squares intro (KA link)

1. Factor $x^2 - 36$ as the product of two binomials.

$\left(a^2 - b^2\right) = (a + b)\,(a - b)$

$x^2 - 36 = x^2 - 6^2 = (x + 6)\,(x - 6)$

2. Factor $4 - x^2$ as the product of two binomials.

$\left(a^2 - b^2\right) = (a + b)\,(a - b)$

$2^2 - x^2 = (2 + x)\,(2 - x)$

Section 5: Difference of squares (KA link)

1. Factor $25 - 64\,y^2$ completely.

$25 - 64\,y^2 = 5^2 - (8\,y)^2 = (5 + 8\,y)\,(5 - 8\,y)$

2. Factor $3\,x^2 - 108$ completely.

$3\,x^2 - 108 = 3\left(x^2 - 36\right) = 3\left(x^2 - 6^2\right) = 3\,(x + 6)\,(x - 6)$

3. Factor $16\,t^2 - 1$ completely.

$16\,t^2 - 1 = (4\,t)^2 - 1^2 = (4\,t + 1)\,(4\,t - 1)$

Section 6: Perfect squares intro (KA link)

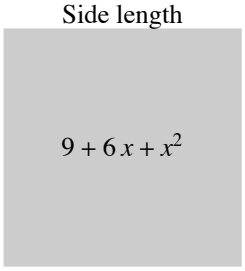
1. Factor $16 - 8\,x + x^2$ as the product of two binomials.

$(a + b)^2 = a^2 + 2\,a\,b + b^2$

$(a - b)^2 = a^2 - 2\,a\,b + b^2$

$16 - 8\,x + x^2 = 4^2 - 2 \cdot 4\,x + x^2 = (4 - x)^2$

2. The square below has an area of $9 + 6\,x + x^2$. What expression represents the length of one side of the square?



$(a + b)^2 = a^2 + 2\,a\,b + b^2$

$(a - b)^2 = a^2 - 2\,a\,b + b^2$

$9 + 6\,x + x^2 = 3^2 + 2 \cdot 3\,x + x^2 = (3 + x)^2$

$A = s^2 \implies s = \sqrt{A} = \sqrt{(3 + x)^2} = 3 + x$

Section 7: Perfect squares (KA link)

1. Factor $1 + 14\,x + 49\,x^2$ completely.

$1 + 14\,x + 49\,x^2 = 1^2 + 2 \cdot 7\,x + (7\,x)^2 = (1 + 7\,x)^2$

2. Factor $2\,w^2 - 44\,w + 242$ completely.

$2\,w^2 - 44\,w + 242 = 2\left(w^2 - 22\,w + 121\right) = 2\left(w^2 - 2 \cdot 11\,w + 11^2\right) = 2\,(w - 11)^2$

3. Factor $64\,x^2 - 160\,x + 100$ completely.

$64\,x^2 - 160\,x + 100 = (8\,x)^2 - 2 \cdot 80\,x + 10^2 = (8\,x - 10)^2$

Section 8: Factor polynomials: special product forms (KA link)

1. Factor $36\,a^2 - 60\,a\,b + 25\,b^2$ completely.

$36\,a^2 - 60\,a\,b + 25\,b^2 = (6\,a)^2 - 2 \cdot 30\,a\,b + (5\,b)^2 = (6\,a - 5\,b)^2$

2. Factor $121\,q^8 - 100$ completely.

$121\,q^8 - 100 = \left(11\,q^4\right)^2 - 10^2 = \left(11\,q^4 + 10\right)\left(11\,q^4 - 10\right)$