

## Exponents and Radicals

### Section 1: Multiply and divide powers (integer exponents) (KA link)

1. Rewrite the following expressions in the form  $b^n$ .

a.  $\frac{y^5}{y^8} = y^{5-8} = y^{-3}$

b.  $\frac{7^{-3}}{7^{15}} = 7^{-3-15} = 7^{-18}$

c.  $\frac{x^3}{x^7} = x^{3-7} = x^{-4}$

d.  $(7^{-3})(7^{15}) = 7^{-3+15} = 7^{12}$

e.  $z^{-9} \cdot z^{-21} = z^{-9+(-21)} = z^{-9-21} = z^{-30}$

f.  $y^{-3} y^2 = y^{-3+2} = y^{-1}$

### Section 2: Powers of products and quotients (integer exponents) (KA link)

1. Simplify the following expressions as much as possible.

a.  $(x^{-5} \cdot y^{-4})^{-6} = x^{(-5)(-6)} y^{(-4)(-6)} = x^{30} y^{24}$

b.  $(x^3 \cdot y^6)^{-3} = x^{3(-3)} y^{6(-3)} = x^{-9} y^{-18}$

2. Select the expression equivalent to  $(4^4 \cdot 5^5)^{-3}$

$$(4^4 \cdot 5^5)^{-3} = 4^{4(-3)} \cdot 5^{5(-3)} = 4^{-12} \cdot 5^{-15} = \frac{1}{4^{12} \cdot 5^{15}}$$

a.  $\frac{5^{15}}{4^{12}}$     b.  $\frac{4^{12}}{5^{15}}$     c.  $\frac{1}{4^{12} \cdot 5^{15}}$

### Section 3: Properties of exponents challenge (integer exponents) (KA link)

1. Explain the mistake in simplifying the following expression, if any:

$$(5^{-3} \cdot 5^{-4})^3 = (5^{-7})^3 = 5^{-4} = \frac{1}{5^4}$$

#### Section 4: Square roots (KA link)

1. Simplify the following square root expressions.

a.  $\sqrt{169} = \sqrt{13^2} = 13$

b.  $\sqrt{225} = \sqrt{15^2} = 15$

c.  $\sqrt{\pi^2} = \pi$

d.  $\sqrt{\sqrt{16}} = \sqrt{\sqrt{4^2}} = \sqrt{4} = 2$

#### Section 5: Cube roots (KA link)

1. Simplify the following cube root expressions.

a.  $\sqrt[3]{216} = \sqrt[3]{36 \cdot 6} = \sqrt[3]{6 \cdot 6 \cdot 6} = \sqrt[3]{6^3} = 6$

b.  $\sqrt[3]{64} = \sqrt[3]{16 \cdot 4} = \sqrt[3]{4 \cdot 4 \cdot 4} = \sqrt[3]{4^3} = 4$

c.  $\sqrt[3]{1728} = \sqrt[3]{144 \cdot 12} = \sqrt[3]{12 \cdot 12 \cdot 12} = \sqrt[3]{12^3} = 12$

d.  $\sqrt[3]{y^3} = y$

#### Section 6: Roots of decimals and fractions (KA link)

1. Simplify the following square root expressions.

a.  $\sqrt{\frac{169}{25}} = \sqrt{\frac{13^2}{5^2}} = \frac{13}{5}$

b.  $\sqrt{.04} = \sqrt{0.2^2} = 0.2$

c.  $\sqrt{1.21} = \sqrt{1.1^2} = 1.1$

d.  $\sqrt{\frac{196}{81}} = \sqrt{\frac{14^2}{9^2}} = \frac{14}{9}$

### Section 7: 4th and 5th roots (KA link)

1. Simplify the following radical expressions.

a.  $\sqrt[4]{1296} = \sqrt[4]{36^2} = \sqrt[4]{6^4} = 6$

b.  $\sqrt[4]{0081} = \sqrt[4]{0.09^2} = \sqrt[4]{0.3^4} = 0.3$

c.  $\sqrt[5]{1024} = \sqrt[5]{2 \cdot 512} = \sqrt[5]{4 \cdot 256} = \sqrt[5]{4 \cdot 4 \cdot 64} = \sqrt[5]{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4} = \sqrt[5]{4^5} = 4$

d.  $\sqrt[5]{\frac{32}{243}} = \sqrt[5]{\frac{2^5}{3^5}} = \frac{2}{3}$

### Section 8: Unit-fraction exponents (KA link)

1. Rewrite the following radical expressions in exponential form.

a.  $\sqrt{t} = t^{1/2}$

b.  $\sqrt[4]{x} = x^{1/4}$

c.  $\sqrt[5]{1/y} = \left(\frac{1}{y}\right)^{1/5} = \frac{1}{y^{1/5}} = y^{-1/5}$

d.  $\sqrt[6]{x/y} = \left(\frac{x}{y}\right)^{1/6} = \frac{x^{1/6}}{y^{1/6}} = x^{1/6} y^{-1/6}$

### Section 9: Fractional exponents (KA link)

1. Rewrite the following radical expressions in exponential form.

a.  $\sqrt{t^3} = (t^3)^{1/2} = t^{3/2}$

b.  $\sqrt[4]{x^5} = (x^5)^{1/4} = x^{5/4}$

c.  $\sqrt[5]{1/y^2} = \left(\frac{1}{y^2}\right)^{1/5} = \frac{1}{y^{2/5}} = y^{-2/5}$

d.  $\sqrt[6]{(x/y)^4} = \left(\frac{x^4}{y^4}\right)^{1/6} = \frac{x^{4/6}}{y^{4/6}} = \frac{x^{2/3}}{y^{2/3}} = x^{2/3} y^{-2/3}$

## Section 10: Rational exponents challenge (KA link)

1. Which expressions are equivalent to  $(w^{-1})^{1/7}$ ? Choose all answers that apply.

$$(w^{-1})^{1/7} = w^{-1/7}$$

a.  $(\sqrt[7]{w})^{-1} = (w^{1/7})^{-1} = w^{-1/7}$

b.  $w^{-1/7}$

c.  $(w^{1/7})^{-1} = w^{-1/7}$

d. None of these.

2. Which expressions are equivalent to  $\sqrt[5]{v^{-3}}$ ? Choose all answers that apply.

$$\sqrt[5]{v^{-3}} = (v^{-3})^{1/5} = v^{-3/5}$$

a.  $\left(\frac{1}{v^3}\right)^{-1/5} = (v^3)^{-1/5} = v^{-3/5}$

b.  $(\sqrt[3]{v})^{-5} = (v^{1/3})^{-5} = v^{-5/3}$

c.  $v^{-3/5}$

d. None of these.

## Section 11: Properties of exponents (rational exponents) (KA link)

1. Rewrite the following expressions in the form  $z^n$ .

a.  $\frac{1}{z^{-1/3}} = z^{1/3}$

b.  $\frac{z^{-1/7}}{z^2} = z^{-1/7} z^{-2} = z^{-1/7} z^{-14/7} = z^{-1/7-14/7} = z^{-15/7}$

c.  $z^{3/5} z^{1/3} = z^{9/15} z^{5/15} = z^{9/15+5/15} = z^{14/15}$

d.  $(z^{-4/3})^{-6/7} = z^{\frac{-4}{3} \cdot \frac{-6}{7}} = z^{8/7}$

## Section 12: Properties of exponents challenge (rational exponents) (KA link)

1. Rewrite the following expressions in the form  $kx^n$ .

a.  $3\sqrt{r} \cdot 5r^{-7/2} = 15r^{1/2}r^{-7/2} = 15r^{1/2-7/2} = 15r^{-6/2} = 15r^{-3}$

b.  $\frac{8\sqrt[3]{w}}{3w^3} = \frac{8}{3} \frac{w^{1/4}}{w^3} = \frac{8}{3} w^{1/4} w^{-3} = \frac{8}{3} w^{1/4-12/4} = \frac{8}{3} w^{-11/4}$

c.  $\frac{y^{-1/2}}{\sqrt[3]{1/y^2}} = \frac{y^{-1/2}}{(y^{-2})^{1/3}} = \frac{y^{-1/2}}{y^{-2/3}} = y^{-1/2} y^{2/3} = y^{-3/6} y^{4/6} = y^{-3/6+4/6} = y^{1/6}$

2. Rewrite the following expressions as a sum of terms, where each term is in the form  $kx^n$ .

a.  $\sqrt[3]{x} \left(2x^3 - \frac{4}{x}\right) = x^{1/3}(2x^3 - 4x^{-1}) = 2x^{1/3}x^3 - 4x^{1/3}x^{-1} = 2x^{1/3+9/3} - 4x^{1/3-3/3} = 2x^{10/3} - 4x^{-2/3}$

b.  $\frac{5x^6-3x^2+x}{\sqrt[3]{x}} = \frac{5x^6}{x^{1/3}} - \frac{3x^2}{x^{1/3}} + \frac{x}{x^{1/3}} = 5x^{18/3-1/3} - 3x^{6/3-1/3} + x^{3/3-1/3} = 5x^{17/3} - 3x^{5/3} + x^{2/3}$

c.  $\frac{2\sqrt{x-x^{1/2}}}{\sqrt[3]{8x}} = \frac{2x^{1/2}}{8^{1/3}x^{1/3}} - \frac{x^{-1/2}}{8^{1/3}x^{1/3}} = x^{3/6-2/6} - \frac{x^{-3/6-2/6}}{2} = x^{1/6} - \frac{x^{-5/6}}{2}$

## Section 13: Evaluate radical expressions challenge (KA link)

1. Evaluate.

a.  $\sqrt[5]{\frac{1}{4}} \cdot \sqrt[5]{128} = \sqrt[5]{\frac{128}{4}} = 32^{1/5} = (2^5)^{1/5} = 2$

b.  $1875^{1/4} \sqrt[4]{\frac{1}{3}} = \left(\frac{1875}{3}\right)^{1/4} = 625^{1/4} = (5^4)^{1/4} = 5$

c.  $\sqrt[3]{-686} \cdot 2^{1/3} = \left(\frac{-686}{2}\right)^{1/3} = (-343)^{1/3} = ((-7)^3)^{1/3} = -7$

d.  $\frac{32^{-29/30}}{32^{-1/6}} = 32^{-29/30} \cdot 32^{1/6} = 32^{-29/30} \cdot 32^{5/30} = 32^{-24/30} = 32^{-4/5} = \frac{1}{(32^{1/5})^4} = \frac{1}{2^4} = \frac{1}{16}$

### Section 14: Simplify square roots (KA link)

1. Simplify (remove all the perfect squares from inside the square root).

a.  $\sqrt{80} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5} = 4 \sqrt{5}$

b.  $\sqrt{363} = \sqrt{3 \cdot 121} = \sqrt{3 \cdot 11 \cdot 11} = 11 \sqrt{3}$

c.  $\sqrt{1764} = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 7 \cdot 7} = 42$

d.  $\sqrt{588} = \sqrt{2 \cdot 2 \cdot 7 \cdot 7 \cdot 3} = 14 \sqrt{3}$

### Section 15: Simplify square roots (variables) (KA link)

1. Simplify (remove all the perfect squares from inside the square root).

a.  $\sqrt{98x^5} = \sqrt{2 \cdot 7 \cdot 7 x^2 x^2 x} = 7x^2 \sqrt{2x}$

b.  $\sqrt{30z^9} = \sqrt{30 z^4 z^4 z} = z^4 \sqrt{30z}$

c.  $\sqrt{32y^{11}} = \sqrt{4 \cdot 4 \cdot 2 y^5 y^5 y} = 4y^5 \sqrt{2y}$

### Section 16: Simplify square-root expressions (KA link)

1. Simplify.

a.  $\sqrt{72x^5y^7} = \sqrt{6 \cdot 6 \cdot 2 x^2 x^2 x y^3 y^3 y} = 6x^2y^3 \sqrt{2xy}$

b.  $\sqrt{32y^{13}z^9} = \sqrt{4 \cdot 4 \cdot 2 y^6 y^6 y z^4 z^4 z} = 4y^6z^4 \sqrt{2yz}$

c.  $\sqrt{3y^4} \cdot \sqrt{7x^3} \cdot \sqrt{15} = y^2 \sqrt{3} x \sqrt{7x} \sqrt{3 \cdot 5} = 3xy^2 \sqrt{35x}$